

# Supersymmetric nonperturbative formulation of the WZ model in lower dimensions

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## Abstract

A nonperturbative formulation of the Wess-Zumino (WZ) model in two and three dimensions is proposed on the basis of momentum-modes truncation. The formulation manifestly preserves full supersymmetry as well as the translational invariance and all global symmetries, while it is shown to be consistent with the expected locality to all orders of perturbation theory. For the two-dimensional WZ model, a well-defined Nicolai map in the formulation provides an interesting algorithm for Monte Carlo simulations.

*Key words:* Supersymmetry, Other nonperturbative techniques, Field theories in dimensions other than four

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## 1 Introduction

In this Letter, we propose a nonperturbative Euclidean formulation of a dimensional reduction of the Wess-Zumino (WZ) model [1] to three and two dimensions.<sup>1</sup> On a nonperturbative formulation of the WZ model, there exist many preceding studies, mostly based on spacetime or spatial lattices [2–45].<sup>2</sup> (For exact renormalization group approaches to the WZ model, see Refs. [48,49].) The desired features of our present proposal are: (I) full supersymmetry (SUSY) as well as the translational invariance and all global

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<sup>1</sup> We assume that the Kähler potential is the flat one,  $\Phi^\dagger\Phi$ , and the superpotential is cubic in the three-dimensional model, so that the model is ultraviolet (UV) finite.

<sup>2</sup> In Sec. 2.2, we will clarify the relation of our proposal to a lattice formulation in Ref. [11] that is based on the SLAC derivative [46,47].

symmetries are manifestly preserved, (II) it is amenable to nonperturbative studies by, for example, Monte Carlo simulations, (III) the formulation of the two-dimensional (2D)  $\mathcal{N} = (2, 2)$  WZ model possesses a well-defined Nicolai map [50,51] (see also Refs. [52,53]). On the other hand, the locality and the reflection positivity are not manifest in our formulation and we will show that there is actually no problem concerning these in lower-dimensional models, at least to all orders of perturbation theory. Therefore, we propose our formulation as a nonperturbative definition of the WZ model in lower dimensions, although its nonperturbative validity still remains to be examined by using, for example, numerical simulations.

Our idea is very simple. The off-shell super multiplets in the WZ model (the chiral and anti-chiral multiplets) are expressed by the chiral and anti-chiral superfields,  $\Phi$  and  $\Phi^\dagger$ .<sup>3</sup> In the momentum space,

$$\begin{aligned}\tilde{\Phi}(p, \theta, \bar{\theta}) &= e^{-\theta\sigma_\mu\bar{\theta}p_\mu} \left[ \tilde{A}(p) + \sqrt{2}\theta\tilde{\psi}(p) + \theta\theta\tilde{F}(p) \right], \\ \tilde{\Phi}^\dagger(p, \theta, \bar{\theta}) &= e^{\theta\sigma_\mu\bar{\theta}p_\mu} \left[ \tilde{A}^*(p) + \sqrt{2}\bar{\theta}\tilde{\psi}(p) + \bar{\theta}\bar{\theta}\tilde{F}^*(p) \right],\end{aligned}\quad (1.1)$$

as they satisfy the chiral constraints,  $\bar{D}_{\dot{\alpha}}\tilde{\Phi}(p) = 0$  and  $D_\alpha\tilde{\Phi}^\dagger(p) = 0$ , where covariant spinor derivatives are given by  $D_\alpha = \partial/\partial\theta^\alpha - \sigma_{\mu\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}p_\mu$  and  $\bar{D}_{\dot{\alpha}} = -\partial/\partial\bar{\theta}^{\dot{\alpha}} + \theta^\alpha\sigma_{\mu\alpha\dot{\alpha}}p_\mu$ . On such off-shell multiplets in the momentum space, SUSY is linearly realized and super transformations generated by

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} + \sigma_{\mu\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}p_\mu, \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - \theta^\alpha\sigma_{\mu\alpha\dot{\alpha}}p_\mu, \quad (1.2)$$

do not mix momentum modes with different momenta. This fact suggests that one can regularize the functional integral of the model by restricting possible momenta of off-shell super multiplets. *Any* restriction on momenta does not break SUSY.<sup>4</sup> From a perspective of the Euclidean rotational symmetry in the infinite volume, it would be preferable to take a rotational invariant restriction

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<sup>3</sup> We follow the notational conventions of Ref. [54], except that we consider Euclidean field theory in terms of momentum modes. (The Fourier transformation is defined by  $\Phi(x, \theta, \bar{\theta}) = \frac{1}{L^d} \sum_p e^{ipx} \tilde{\Phi}(p, \theta, \bar{\theta})$ .) The Euclidean time  $x_0$  is defined from

the Lorentzian time  $x^0$  by  $x^0 \rightarrow -ix_0$  and we set  $\sigma_0 \equiv \bar{\sigma}_0 \equiv \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$ . Summation

over repeated indices is always meant and the Greek index  $\mu$  runs from 0 to  $d-1$ , where  $d \leq 4$  is the spacetime dimension. Although we consider the system with a single chiral multiplet for notational simplicity, the generalization to cases with multi chiral multiplets is straightforward.

<sup>4</sup> It is crucial that we restrict the momentum of off-shell super multiplets on which SUSY is linearly realized. If the super transformations are non-linear in field variables, such as in supersymmetric gauge theories in the Wess-Zumino gauge, super transformations mix modes with different momenta and restriction on possible momenta breaks SUSY. In this aspect, our formulation differs from a formulation of

such as  $p^2 \equiv p_\mu p_\mu \leq \Lambda^2$ , where  $\Lambda$  is an UV cutoff. Thus, one may define a regularized partition function of the WZ model by

$$\mathcal{Z} \equiv \int \prod_{p^2 \leq \Lambda^2} \left[ d\tilde{A}(p) d\tilde{A}^*(p) d\tilde{F}(p) d\tilde{F}^*(p) \prod_{\alpha=1}^2 d\tilde{\psi}_\alpha(p) \prod_{\dot{\alpha}=\dot{1}}^{\dot{2}} d\tilde{\bar{\psi}}_{\dot{\alpha}}(p) \right] e^{-S}. \quad (1.3)$$

To make this expression fully well-defined, we may assume that the system is put in a Euclidean box of size  $L$  and the momentum  $p$  is discrete,  $p_\mu = (2\pi/L)n_\mu$ , where  $n_\mu \in \mathbb{Z}$ . The Euclidean action  $S$  in the momentum space reads

$$S \equiv - \int d^4\theta (\tilde{\Phi}^\dagger * \tilde{\Phi})(0, \theta, \bar{\theta}) - \int d^2\theta W(\tilde{\Phi})(0, \theta, \bar{\theta}) - \text{h.c.}, \quad (1.4)$$

where the symbol  $*$  denotes the convolution

$$(\tilde{\Phi}_1 * \tilde{\Phi}_2)(p, \theta, \bar{\theta}) \equiv \frac{1}{L^d} \sum_q \tilde{\Phi}_1(q, \theta, \bar{\theta}) \tilde{\Phi}_2(p - q, \theta, \bar{\theta}), \quad (1.5)$$

and the product in the superpotential  $W(\tilde{\Phi})(0, \theta, \bar{\theta})$  in Eq. (1.4) is defined by repeated applications of this convolution rule. We see that action (1.4) is invariant under a multiplication of operators (1.2) on each field variables, because the sum of momenta in action (1.4) is zero, corresponding to the translational invariance.

Prescription (1.3) thus manifestly preserves SUSY as well as the translational invariance. All global symmetries of the action (such as the  $R$ -symmetry) are also manifest. Specifically, one can derive Ward-Takahashi identities associated with these symmetries within a regularized framework. Since definition (1.3) does not modify the spinor-space structure of the WZ model, one may repeat the proof of perturbative non-renormalization theorems [56–58] in this regularized framework (cf. Ref. [54]). One may also repeat the argument of Ref. [59] because the holomorphy is manifestly preserved. Furthermore, prescription (1.3) is amenable to nonperturbative studies by, for example, Monte Carlo simulations, because Eq. (1.3) is a finite-dimensional integral for finite  $\Lambda$  and  $L$ .

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supersymmetric gauge theories in Ref. [55], in which SUSY is expected to be exact only in the limit that the momentum cutoff is removed. Note that the present model has no gauge symmetry; it is clear that any restriction on possible momenta breaks local gauge invariance.

## 2 Locality

### 2.1 Locality and finiteness

The above description sounds too good to be true. Actually, it is not clear whether definition (1.3) (in the limit that the UV and IR cutoffs are removed,  $\Lambda \rightarrow \infty$  and  $L \rightarrow \infty$ ) is consistent with the expected locality in the target theory. The reflection positivity is a related issue. Prescription (1.3) differs from the conventional momentum cutoff in perturbative Feynman integrals, with which the locality would be obvious. The point is that the restriction  $p^2 \leq \Lambda^2$  on integration variables in Eq. (1.3) could introduce non-smooth dependence of a Feynman integral on external momenta; such dependence could not be interpreted as an insertion of local operators. See the analysis in Appendix.<sup>5</sup>

Now, if the model is massive,  $W(\Phi) = (1/2)m\Phi^2 + \dots$ , the free super propagators are given by

$$\langle \tilde{\Phi}(p, \theta, \bar{\theta}) \tilde{\Phi}^\dagger(q, \theta', \bar{\theta}') \rangle = \frac{1}{16} \bar{D}^2 D^2 \frac{1}{p^2 + |m|^2} (2\pi)^d \delta(p + q) \delta(\theta - \theta') \delta(\bar{\theta} - \bar{\theta}'), \quad (2.1)$$

$$\langle \tilde{\Phi}^\dagger(p, \theta, \bar{\theta}) \tilde{\Phi}(q, \theta', \bar{\theta}') \rangle = \frac{1}{16} D^2 \bar{D}^2 \frac{1}{p^2 + |m|^2} (2\pi)^d \delta(p + q) \delta(\theta - \theta') \delta(\bar{\theta} - \bar{\theta}'), \quad (2.2)$$

$$\langle \tilde{\Phi}(p, \theta, \bar{\theta}) \tilde{\Phi}(q, \theta', \bar{\theta}') \rangle = \frac{1}{4} \bar{D}^2 \frac{m^*}{p^2 + |m|^2} (2\pi)^d \delta(p + q) \delta(\theta - \theta') \delta(\bar{\theta} - \bar{\theta}'), \quad (2.3)$$

$$\langle \tilde{\Phi}^\dagger(p, \theta, \bar{\theta}) \tilde{\Phi}^\dagger(q, \theta', \bar{\theta}') \rangle = \frac{1}{4} D^2 \frac{m}{p^2 + |m|^2} (2\pi)^d \delta(p + q) \delta(\theta - \theta') \delta(\bar{\theta} - \bar{\theta}'), \quad (2.4)$$

where the momentum contained in  $D_\alpha$  and  $\bar{D}_{\dot{\alpha}}$  is  $p$ . In prescription (1.3), the functional integral is defined in terms of momentum modes. One may then define the field variable in the real space by  $\Phi(x, \theta, \bar{\theta}) = \int_{p^2 \leq \Lambda^2} \frac{d^d p}{(2\pi)^d} e^{ipx} \tilde{\Phi}(p, \theta, \bar{\theta})$ . Then free super propagators in the real space are proportional to

$$\begin{aligned} & \int_{p^2 \leq \Lambda^2} \frac{d^d p}{(2\pi)^d} \frac{e^{ip(x-y)}}{p^2 + |m|^2} \\ &= \frac{(2\pi)^{d/2}}{|x-y|^{d/2-1}} \left[ |m|^{d/2-1} K_{d/2-1}(|m||x-y|) - \int_{\Lambda}^{\infty} dp \frac{p^{d/2} J_{d/2-1}(p|x-y|)}{p^2 + |m|^2} \right], \end{aligned} \quad (2.5)$$

<sup>5</sup> In this subsection and in Appendix, where the issue of locality is addressed, we set  $L \rightarrow \infty$  because the notion of locality becomes transparent only in this limit.

where  $J_\nu(z)$  ( $K_\nu(z)$ ) denotes the (modified) Bessel function. In the right-hand side, the first term is the standard massive propagator which dumps exponentially  $\sim e^{-|m||x-y|}$ . The second term is the cutoff effect and its amplitude dumps only in the inverse powers of  $|x-y|$ . Therefore, when  $\Lambda$  is kept fixed, the second term dominates the first for  $|x-y|$  large. Nevertheless, since the second term is integrable at  $p \rightarrow \infty$ , the second term with  $|x-y|$  kept fixed vanishes as  $\Lambda \rightarrow \infty$ . Free propagators in the real space thus restore the expected locality for  $\Lambda \rightarrow \infty$ .

Next, we consider the effect of interaction. In prescription (1.3), only momentum modes with  $p^2 \leq \Lambda^2$  appear and, in perturbation theory, this restriction can be taken into account by substituting all factors  $1/(p^2 + |m|^2)$  in Eqs. (2.1)–(2.4) by  $\Theta(\Lambda^2 - p^2)/(p^2 + |m|^2)$ , where  $\Theta(x)$  denotes the step function. The locality is not obvious in general with this prescription as illustrated in Appendix. Nevertheless, if a convergence property of a Feynman integral is good enough, the value of the Feynman integral must be independent of the regularization as  $\Lambda \rightarrow \infty$ ; then the issue of locality and reflection positivity should not matter. Since our formulation preserves manifest SUSY, we expect a better convergence property of Feynman integrals compared with formulations which do not have manifest SUSY.<sup>6</sup>

Since SUSY is manifest with prescription (1.3), one can determine the superficial degrees of divergence on the basis of the super Feynman rule (see Ref. [54]). In the WZ model in  $d$  dimensions, the superficial degrees of divergence  $\omega(\Gamma)$  of a super diagram  $\Gamma$  is given by (cf. Sec. 6.6 of Ref. [61])

$$\omega(\Gamma) = d - 2 - \frac{1}{2}(d-2)E + \sum_i \left[ \frac{1}{2}(d-2)i + 1 - d \right] V_i - C, \quad (2.6)$$

where  $E$  denotes the number of external lines,  $V_i$  the number of the  $\Phi^i$ -type interaction vertex and  $C$  the number of the (anti-)chiral propagators, Eqs. (2.3) and (2.4).

For the four-dimensional (4D) system,  $d = 4$ , the perturbative renormalizability requires the superpotential is cubic  $W(\Phi) = (1/2)m\Phi^2 + (1/3)g\Phi^3$ . Then Eq. (2.6) yields  $\omega(\Gamma) = 2 - E - C$  and we see that only two-point functions may logarithmically diverge (tadpoles, for which  $E = 1$ , identically vanish owing to the non-renormalization theorem; see Ref. [54]). For such divergent diagrams, the locality is not obvious. In fact, as explained in Appendix, we could neither prove nor disprove consistency of prescription (1.3) with the expected locality beyond the one-loop level. Thus we must admit that the validity of prescription (1.3) is not clear for 4D WZ model.

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<sup>6</sup> If SUSY were not manifest, even perturbation theory in a dimensional reduction of the WZ model to one dimension would suffer from logarithmic divergences (see Ref. [60]).

For the three-dimensional (3D) system,  $d = 3$ , if the superpotential is cubic  $W(\Phi) = (1/2)m\Phi^2 + (1/3)g\Phi^3$ , we have  $\omega(\Gamma) = 1 - (1/2)E - (1/2)V_3 - C$  and, since again the tadpoles identically vanish owing to SUSY, all Feynman diagrams have strictly negative superficial degrees of divergence.

For the two-dimensional (2D) system,  $d = 2$ , for any (polynomial) superpotential, we have  $\omega(\Gamma) = -\sum_i V_i - C$  and again all Feynman diagrams have strictly negative superficial degrees of divergence.

The above counting shows that, in 3D  $\mathcal{N} = 2$  WZ model with the cubic superpotential and in 2D  $\mathcal{N} = (2, 2)$  WZ model with arbitrary superpotential, all 1PI diagrams have strictly negative superficial degrees of divergence. Combined this with the power-counting theorem [62,63], we see that all Feynman integrals in these lower-dimensional models are *absolutely convergent*. We then intuitively expect that, owing to this good convergence property, the correct (finite) value of Feynman diagrams is reproduced with prescription (1.3) in the  $\Lambda \rightarrow \infty$  limit. Then there will be no need to worry about the locality and the reflection positivity for  $\Lambda \rightarrow \infty$ .

This natural expectation is rigorously confirmed by the following

**Lemma 1** *For any Feynman integral*

$$I_F(p) \equiv \int d^d k_1 \cdots d^d k_L \mathcal{I}_F(k, p), \quad (2.7)$$

where  $k_i$  are loop momenta and  $p$  collectively denotes external momenta, that is absolutely convergent  $\int d^d k_1 \cdots d^d k_L |\mathcal{I}_F(k, p)| < \infty$ , we have

$$\lim_{\Lambda \rightarrow \infty} \int d^d k_1 \cdots d^d k_L \mathcal{I}'_F(k, p; \Lambda) = I_F(p) \quad (2.8)$$

for any fixed  $p$ , where  $\mathcal{I}'_F(k, p; \Lambda)$  is a modified integrand that is defined by substituting all propagators  $1/(\ell_i^2 + |m|^2)$  in the original integrand  $\mathcal{I}_F(k, p)$  by  $\Theta(\Lambda^2 - \ell_i^2)/(\ell_i^2 + |m|^2)$ .

**PROOF.** From the definition of  $\mathcal{I}'_F(k, p; \Lambda)$ ,

$$\begin{aligned} & \left| \int d^d k_1 \cdots d^d k_L \mathcal{I}_F(k, p) - \int d^d k_1 \cdots d^d k_L \mathcal{I}'_F(k, p; \Lambda) \right| \\ & \leq \int d^d k_1 \cdots d^d k_L |\mathcal{I}_F(k, p) - \mathcal{I}'_F(k, p; \Lambda)| \\ & = \int d^d k_1 \cdots d^d k_L |\mathcal{I}_F(k, p)| \left[ 1 - \Theta(\Lambda^2 - \ell_1^2) \cdots \Theta(\Lambda^2 - \ell_N^2) \right], \end{aligned} \quad (2.9)$$

where  $N$  denotes the number of propagators and propagators' momenta  $\ell_i$  are linear combinations of  $k$  and  $p$ . For fixed  $p$ , for sufficiently large  $\Lambda$ , there exists a region containing the origin of  $\mathbb{R}^{Ld}$  of size  $\Lambda'$ ,  $B(\Lambda') \equiv \{k_{i\mu} \in \mathbb{R}^{Ld} \mid$

$|k_{i\mu}| \leq \Lambda'/2\}$ , such that  $\Theta(\Lambda^2 - \ell_1) \cdots \Theta(\Lambda^2 - \ell_N^2) = 1$  for  $k \in B(\Lambda')$ . Setting  $\mathbb{R}^{Ld} = B(\Lambda') \cup \bar{B}(\Lambda')$ , since  $1 - \Theta(\Lambda^2 - \ell_1) \cdots \Theta(\Lambda^2 - \ell_N^2) = 0$  for  $k \in B(\Lambda')$  and  $|1 - \Theta(\Lambda^2 - \ell_1) \cdots \Theta(\Lambda^2 - \ell_N^2)| \leq 1$ , we have

$$\text{Eq. (2.9)} \leq \int_{\bar{B}(\Lambda')} d^d k_1 \cdots d^d k_L |\mathcal{I}_F(k, p)|. \quad (2.10)$$

We then take the  $\Lambda \rightarrow \infty$  limit on the both sides of Eq. (2.10). The most left-hand side of (2.9) becomes  $|I_F(p) - \lim_{\Lambda \rightarrow \infty} \int d^d k_1 \cdots d^d k_L \mathcal{I}'_F(k, p; \Lambda)|$ . In the right-hand side of Eq. (2.10), we may then take the  $\Lambda' \rightarrow \infty$  limit and this leads to  $\lim_{\Lambda' \rightarrow \infty} \int_{\bar{B}(\Lambda')} d^d k_1 \cdots d^d k_L |\mathcal{I}_F(k, p)| = 0$  because integral (2.7) is absolutely convergent. This shows Eq. (2.8).

To summarize, to all orders of perturbation theory, definition (1.3) provides a valid formulation of 3D  $\mathcal{N} = 2$  WZ model with the cubic superpotential and of 2D  $\mathcal{N} = (2, 2)$  WZ model with arbitrary superpotential.<sup>7</sup> We thus propose to use Eq. (1.3) as a nonperturbative definition of these models. As already noted, all symmetries in the target theory are manifest and, moreover, the formulation is amenable to nonperturbative Monte Carlo simulations.

## 2.2 Relation to a lattice formulation based on the SLAC derivative

As noted in Introduction, the SUSY invariance holds with any restriction on possible momenta of super multiplets. For example, since the rotational symmetry is in any case broken in a finite box, one may adopt a “cubic” restriction  $-\Lambda \leq p_\mu \leq \Lambda$  for all  $\mu$ , rather than the “spherical one”  $p^2 \leq \Lambda^2$ , and

$$\mathcal{Z}' \equiv \int \prod_{-\Lambda \leq p_\mu \leq \Lambda} \left[ d\tilde{A}(p) d\tilde{A}^*(p) d\tilde{F}(p) d\tilde{F}^*(p) \prod_{\alpha=1}^2 d\tilde{\psi}_\alpha(p) \prod_{\dot{\alpha}=1}^2 d\tilde{\psi}_{\dot{\alpha}}(p) \right] e^{-S}, \quad (2.11)$$

where the action  $S$  is again given by Eq. (1.4). This prescription shares desired features with Eq. (1.3), such as SUSY is manifest and the locality is restored in the  $\Lambda \rightarrow \infty$  limit for lower-dimensional models (to all orders of perturbation theory; Lemma 1 can appropriately be modified for the cubic momentum restriction above). Prescription (2.11) is, however, nothing but a lattice formulation of the WZ model in Ref. [11] that is based on the SLAC lattice derivative [46,47].<sup>8</sup> In fact, if one expresses the lattice formulation in Ref. [11]

<sup>7</sup> Power-counting shows that if we generalize the Kähler potential to an arbitrary real function  $\Phi^\dagger \Phi \rightarrow K(\Phi^\dagger, \Phi)$ , new logarithmic divergences may appear and the present argument does not apply.

<sup>8</sup> See also Refs. [15,16] and Refs. [37,41,43] for related formulations.

(Eqs. (2.10) and (2.27) there, after the Wick rotation) in terms of momentum modes  $\Phi(x, \theta, \bar{\theta}) = \frac{1}{L^d} \sum_p e^{ipx} \tilde{\Phi}(p, \theta, \bar{\theta})$ , where  $x$  denotes the lattice point, one ends up with Eq. (2.11) with the identification  $\Lambda \equiv \pi/a$  ( $a$  is the lattice spacing).

Since two prescriptions (1.3) and (2.11) should be essentially equivalent for  $\Lambda \rightarrow \infty$ , our proposal (1.3) is basically equivalent to the lattice formulation in Ref. [11] on the basis of the SLAC derivative.<sup>9</sup> The SLAC derivative is not usually adopted in lattice (gauge) theory, because the locality could be violated [64]. See also Refs. [2, 65]. This is also the case with our prescription for 4D WZ model; as discussed in Appendix, the consistency of prescription (1.3) with locality is not clear for 4D WZ model. However, as we have discussed so far, the prescription can be consistent with the locality in lower-dimensional models and we can expect the same for prescription (2.11).

Thus, from this perspective, our contribution in the present Letter is merely in that we gave a strong affirmative argument for the applicability of the formulation in Ref. [11] to 3D  $\mathcal{N} = 2$  and 2D  $\mathcal{N} = (2, 2)$  WZ models, dimensional reductions of the original 4D  $\mathcal{N} = 1$  WZ model. On the other hand, we have to say that its validity for 4D WZ model itself is still not clear, unfortunately.

### 3 2D $\mathcal{N} = (2, 2)$ WZ model

2D  $\mathcal{N} = (2, 2)$  WZ model is interesting in its own right, because, for example, it provides the Landau-Ginsburg model for  $\mathcal{N} = (2, 2)$  superconformal field theory [66–70]. In the dimensional reduction from four to two dimensions, we set  $\mu = 0$  and  $\mu = 3$  directions unreduced. Then, from Eq. (1.4), in terms of component fields in Eq. (1.1), we have

$$S = \frac{1}{L^2} \sum_p \left[ 4p_z \tilde{A}^*(-p) p_{\bar{z}} \tilde{A}(p) - \tilde{F}^*(-p) \tilde{F}(p) \right. \\ \left. - \tilde{F}^*(-p) * W'(\tilde{A})^*(p) - \tilde{F}(-p) * W'(\tilde{A})(p) \right. \\ \left. + (\tilde{\psi}_1, \tilde{\psi}_2)(-p) \begin{pmatrix} 2ip_z & W''(\tilde{A})^{**} \\ W''(\tilde{A})^* & 2ip_{\bar{z}} \end{pmatrix} \begin{pmatrix} \tilde{\psi}_1 \\ \tilde{\psi}_2 \end{pmatrix}(p) \right], \quad (3.1)$$

where we have defined  $p_z \equiv (1/2)(p_0 - ip_3)$  and  $p_{\bar{z}} \equiv (1/2)(p_0 + ip_3)$ ; the product in  $W''(\tilde{A})$  has been defined by repeated applications of convolution (1.5). Since SUSY is manifest in our formulation, we could repeat the argument in,

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<sup>9</sup> Our description in a previous version of this Letter on the relation to SLAC-derivative-based lattice formulations was inadequate. We would like to thank Yoshio Kikukawa for clarifying discussions on this point.



for example, Sec. 2 of Ref. [71]. For the argument there, important fermionic symmetries are (in the notation of Ref. [54])

$$\begin{aligned}
\bar{Q}_1 \tilde{\psi}_1(p) &= -2\sqrt{2}ip_z \tilde{A}(p), & \bar{Q}_1 \tilde{A}(p) &= 0, \\
\bar{Q}_1 \tilde{F}(p) &= -2\sqrt{2}ip_z \tilde{\psi}_2(p), & \bar{Q}_1 \tilde{\psi}_2(p) &= 0, \\
\bar{Q}_1 \tilde{A}^*(p) &= \sqrt{2} \tilde{\psi}_1(p), & \bar{Q}_1 \tilde{\psi}_1(p) &= 0, \\
\bar{Q}_1 \tilde{\psi}_2(p) &= -\sqrt{2} \tilde{F}^*(p), & \bar{Q}_1 \tilde{F}^*(p) &= 0,
\end{aligned} \tag{3.2}$$

and

$$\begin{aligned}
\bar{Q}_2 \tilde{\psi}_2(p) &= -2\sqrt{2}ip_z \tilde{A}(p), & \bar{Q}_2 \tilde{A}(p) &= 0, \\
\bar{Q}_2 \tilde{F}(p) &= 2\sqrt{2}ip_z \tilde{\psi}_1(p), & \bar{Q}_2 \tilde{\psi}_1(p) &= 0, \\
\bar{Q}_2 \tilde{A}^*(p) &= \sqrt{2} \tilde{\psi}_2(p), & \bar{Q}_2 \tilde{\psi}_2(p) &= 0, \\
\bar{Q}_2 \tilde{\psi}_1(p) &= \sqrt{2} \tilde{F}^*(p), & \bar{Q}_2 \tilde{F}^*(p) &= 0.
\end{aligned} \tag{3.3}$$

These nilpotent symmetries imply that, among correlation functions of scalar fields  $\tilde{A}(p)$ , only those of zero momentum modes  $\tilde{A}(0)$  can be nontrivial. This follows from the fact that  $\tilde{A}(p)$  with  $p \neq 0$  are  $\bar{Q}_1$  or  $\bar{Q}_2$  exact and  $\tilde{A}(p)$  are closed under  $\bar{Q}_1$  and  $\bar{Q}_2$ .<sup>10</sup> Moreover, since the anti-holomorphic part of the superpotential is  $\bar{Q}_1$  or  $\bar{Q}_2$  exact, correlation functions of  $\tilde{A}(0)$  depend on parameters in the superpotential only holomorphically (i.e., they depend on  $g$  but not on  $g^*$ ). It is interesting that our prescription provides a solid basis for these arguments which assume a supersymmetric regularization. In the context of the Landau-Ginsburg model for the superconformal field theory, one has to consider the massless (or critical) limit. Since in this limit perturbation theory suffers from severe infrared divergences, it must be important to formulate the system nonperturbatively.

It is well known that 2D  $\mathcal{N} = (2, 2)$  WZ model possesses the Nicolai map [50,51] which is a mapping that “trivializes” the functional integral.<sup>11</sup> Action (3.1)

<sup>10</sup> The Witten index of the present system (with a single chiral multiplet) is  $n$ , when the superpotential  $W(\phi)$  is an  $(n+1)$ -th order polynomial [72,73]. Thus, for  $n \geq 1$ , SUSY cannot be spontaneously broken.

<sup>11</sup> For 2D  $\mathcal{N} = (2, 2)$  WZ model (on a 2D cylinder), a regularized Hermitian supercharge and an associated Hamiltonian have been constructed and the existence of their finite limit (as the UV cutoff goes to infinity) has rigorously been proven [74,75]. Although on general grounds we expect that if the theory exists the limit that the UV cutoff is removed is unique, to show the equivalence of the present prescription to the construction of Refs. [74,75] is far beyond the scope of this Letter. We expect that the Nicolai map provides a useful clue to show such a nonperturbative finiteness in our approach.

can be rewritten as

$$S = \frac{1}{L^2} \sum_p \left[ -\tilde{G}^*(-p)\tilde{G}(p) - \tilde{G}^*(-p)\tilde{N}(p) - \tilde{G}(-p)\tilde{N}^*(p) \right. \\ \left. + (\tilde{\psi}_1, \tilde{\psi}_2)(-p) \begin{pmatrix} 2ip_z & W''(\tilde{A})^{**} \\ W''(\tilde{A})^* & 2ip_{\bar{z}} \end{pmatrix} \begin{pmatrix} \tilde{\psi}_1 \\ \tilde{\psi}_2 \end{pmatrix}(p) \right], \quad (3.4)$$

where  $\tilde{G}(p) \equiv \tilde{F}(p) - 2ip_z\tilde{A}(p)$  and  $\tilde{G}^*(p) \equiv \tilde{F}^*(p) - 2ip_{\bar{z}}\tilde{A}^*(p)$  are shifted auxiliary fields and the combinations  $\tilde{N}(p)$  and  $\tilde{N}^*(p)$  define the Nicolai map

$$\begin{aligned} & \{\tilde{A}(p), \tilde{A}^*(p)\} \\ & \mapsto \{\tilde{N}(p) \equiv 2ip_z\tilde{A}(p) + W'(\tilde{A})^*(p), \tilde{N}^*(p) \equiv 2ip_{\bar{z}}\tilde{A}^*(p) + W'(\tilde{A})(p)\}. \end{aligned} \quad (3.5)$$

From Eq. (3.4), one sees that the fermion determinant is precisely the Jacobian associated with change of integration variables (3.5). Thus, after the integration over the fermion field, the bosonic integration variables become  $\{\tilde{N}(p), \tilde{N}^*(p)\}$ . Moreover, the action  $S$  becomes Gaussian in  $\{\tilde{N}(p), \tilde{N}^*(p)\}$  after the integration over the auxiliary fields  $\tilde{G}(p)$  and  $\tilde{G}^*(p)$ . In this way, the functional integral is trivialized by map (3.5).

Note that, in functional integral (1.3), the momentum of auxiliary fields  $\tilde{G}(p)$  and  $\tilde{G}^*(p)$  is restricted to  $p^2 \leq \Lambda^2$ . Therefore, in Eq. (3.4), only  $\tilde{N}(p)$  and  $\tilde{N}^*(p)$  with  $p^2 \leq \Lambda^2$  appear. That is, in prescription (1.3), Nicolai map (3.5) is a mapping from  $\{\tilde{A}(p), \tilde{A}^*(p)\}$  to  $\{\tilde{N}(p), \tilde{N}^*(p)\}$ , both are subject of identical momentum restriction  $p^2 \leq \Lambda^2$ . In this sense, the Nicolai map is well-defined with prescription (1.3).

The existence of the Nicolai map provides a quite interesting simulation algorithm for the present system. See Ref. [20] for actual implementation of this idea in a discretized real space. One first generates a set of Gaussian random numbers with the unit covariance; this gives a configuration of  $\{\tilde{N}(p), \tilde{N}^*(p)\}$ . Then one inverts Nicolai map (3.5) by numerical means. There may exist several inverse images and one must in principle find all of them. This provides configuration(s) of  $\{\tilde{A}(p), \tilde{A}^*(p)\}$ . Repeating these steps, one obtains a statistical ensemble of  $\{\tilde{A}(p), \tilde{A}^*(p)\}$ . Correlation functions of the fermion field can also be obtained from bosonic ones without inversion (by assuming that SUSY is not spontaneously broken). A great advantage of this algorithm is that, compared with conventional methods based on the Markov process, there is (in principle) *no* autocorrelation between configurations in the obtained ensemble.<sup>12</sup> Another important point to note is that the fermion determinant

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<sup>12</sup> We would like thank Martin Lüscher for bringing our attention to this point. In Ref. [76], an algorithm on the basis of a map that (approximately) trivializes the functional integral in lattice gauge theory has been constructed, aiming at avoiding

in the present system is generally complex and thus conventional methods may fail owing to the sign problem, while the algorithm based on the Nicolai map seems to be free from this difficulty. In the near future, we hope to carry out Monte Carlo simulations of 2D  $\mathcal{N} = (2, 2)$  WZ model on the basis of the Nicolai map in the present momentum-space formulation.

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## Note added

After completing this work, we became aware of a paper by Georg Bergner [77] in which a lattice Monte Carlo simulation of the SUSY quantum mechanics (QM) [72] was carried out on the basis of a “full supersymmetric model”. This lattice formulation is just the supersymmetric lattice formulation of Ref. [11] applied to SUSY QM. Since SUSY QM is UV finite with a supersymmetric regularization, a variant of Lemma 1 ensures the restoration of locality in this formulation to all orders of perturbation theory.

## A Locality in 4D $\mathcal{N} = 1$ WZ model

Owing to Lemma 1 in the main text, UV (absolutely) convergent Feynman integrals are reproduced in prescription (1.3) with  $\Lambda \rightarrow \infty$ , and thus it suffices to consider UV diverging Feynman diagrams. In the one-loop level, the unique UV diverging 1PI super diagram is the two-point function of  $\tilde{\Phi}$  and  $\tilde{\Phi}^\dagger$ . (The one-loop 1PI functions of  $\tilde{\Phi}$  (or  $\tilde{\Phi}^\dagger$ ) alone identically vanish owing to manifest SUSY; cf. Ref. [54].) With the present prescription, the contribution of this super diagram to the 1PI effective action is

$$S_{\text{eff}} = - \int d^4\theta \, 2|g|^2 \int \frac{d^4p}{(2\pi)^4} \tilde{\Phi}^\dagger(-p, \theta, \bar{\theta}) \tilde{\Phi}(p, \theta, \bar{\theta}) I_F(p), \quad (\text{A.1})$$

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the critical slowing down.

where the one-loop Feynman integral is given by

$$I_F(p) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{\Theta(\Lambda^2 - k^2)}{k^2 + |m|^2} \frac{\Theta(\Lambda^2 - (k+p)^2)}{(k+p)^2 + |m|^2} \quad (\text{A.2})$$

$$= \int \frac{d^4k}{(2\pi)^4} \Theta(\Lambda^2 - k^2) \Theta(\Lambda^2 - (k+p)^2) \frac{k^2 - (k+p)^2}{(k^2 + |m|^2)^2 [(k+p)^2 + |m|^2]} \quad (\text{A.3})$$

$$+ \int \frac{d^4k}{(2\pi)^4} \Theta(\Lambda^2 - k^2) \frac{1}{(k^2 + |m|^2)^2} \quad (\text{A.4})$$

$$- \int \frac{d^4k}{(2\pi)^4} \Theta(\Lambda^2 - k^2) [1 - \Theta(\Lambda^2 - (k+p)^2)] \frac{1}{(k^2 + |m|^2)^2}. \quad (\text{A.5})$$

In the second equality, we have re-organized the integrand to address the locality. First, the integral in Eq. (A.3) is absolutely convergent even without the regularization factor  $\Theta(\Lambda^2 - k^2)\Theta(\Lambda^2 - (k+p)^2)$  and, according to Lemma 1, we may simply discard the factor  $\Theta(\Lambda^2 - k^2)\Theta(\Lambda^2 - (k+p)^2) \rightarrow 1$  in the limit  $\Lambda \rightarrow \infty$ . Therefore, in this limit, Eq. (A.3) becomes nothing but the finite part of the Feynman integral which is given by the BPHZ subtraction scheme applied to the logarithmically divergent integral in the original un-regularized theory. Next, Eq. (A.4)  $\xrightarrow{\Lambda \rightarrow \infty} (1/16\pi^2) \ln[\Lambda^2/(e|m|^2)]$  is an ultraviolet diverging part that would be obtained in the conventional momentum-cutoff regularization. This is a part subtracted by a local counterterm (the wave function renormalization, in the present case) in the BPHZ renormalization.

We have thus observed that, in the  $\Lambda \rightarrow \infty$  limit, Eqs. (A.3) and (A.4) reproduce the correct finite part and a divergent local term corresponding to the BPHZ subtraction scheme. These two terms are thus consistent with the expected locality. On the other hand, if Eq. (A.5) does survive in the  $\Lambda \rightarrow \infty$  limit, the expected locality would be violated because Eq. (A.5) could not be a smooth function of the external momentum  $p$ ; in other words, Eq. (A.5) could not be interpreted by an insertion of local operators.

To estimate Eq. (A.5), we note that the factor  $\Theta(\Lambda^2 - k^2) [1 - \Theta(\Lambda^2 - (k+p)^2)]$  is non-zero only in a region which is sandwiched in between two 3-spheres with the radius  $\Lambda$ , one has its center at  $k = 0$  and another at  $k = -p$ . The 4-volume of this region  $\mathcal{V}(p; \Lambda)$  is given by

$$\begin{aligned} \mathcal{V}(p; \Lambda) &\equiv \frac{1}{48\pi^3} \left[ |p| \left( \Lambda^2 - \frac{p^2}{4} \right)^{3/2} + \frac{3}{2} \Lambda^2 |p| \left( \Lambda^2 - \frac{p^2}{4} \right)^{1/2} + 3\Lambda^4 \arcsin \left( \frac{|p|}{2\Lambda} \right) \right]. \end{aligned} \quad (\text{A.6})$$

On the other hand, we have simple bounds on the factor  $1/(k^2 + |m|^2)^2$

in Eq. (A.5), from the consideration of the integration region,

$$\frac{1}{(\Lambda^2 + |m|^2)^2} \leq \frac{1}{(k^2 + |m|^2)^2} \leq \frac{1}{[(\Lambda - |p|)^2 + |m|^2]^2}. \quad (\text{A.7})$$

Therefore,

$$|\text{Eq. (A.5)}| \leq \frac{\mathcal{V}(p; \Lambda)}{[(\Lambda - |p|)^2 + |m|^2]^2} \xrightarrow{\Lambda \rightarrow \infty} 0, \quad (\text{A.8})$$

when the external momentum  $p$  is kept fixed in the limit, because  $\mathcal{V}(p; \Lambda) = O(\Lambda^3)$  in such a limit. Eq. (A.5) therefore vanishes in the  $\Lambda \rightarrow \infty$  limit and the expected locality is reproduced. This shows that prescription (1.3) is consistent with the locality at least in the one-loop level.

In the two-loop level, however, the situation is much worse because there exist diagrams in which the external momentum  $p$  in Eq. (A.2) becomes a loop momentum that can be as large as the cutoff  $\Lambda$ . The most singular two-loop contribution to the effective action turns to be

$$S_{\text{eff}} = \int d^4\theta 8|g|^4 \int \frac{d^4q}{(2\pi)^4} \tilde{\Phi}^\dagger(-q, \theta, \bar{\theta}) \tilde{\Phi}(q, \theta, \bar{\theta}) \\ \times \int \frac{d^4p}{(2\pi)^4} \frac{\Theta(\Lambda^2 - p^2)}{(p^2 + |m|^2)^2} \frac{(p+q)^2 \Theta(\Lambda^2 - (p+q)^2)}{(p+q)^2 + |m|^2} I_F(p). \quad (\text{A.9})$$

We would be happy, if last term (A.5) does not contribute in the limit  $\Lambda \rightarrow \infty$ , when Eq. (A.2) is substituted in Eq. (A.9). Otherwise, since the effect of Eq. (A.5) could not be interpreted as an insertion of local operators, the expected locality would be broken. Now, we first note that  $|\text{Eq. (A.5)}| \geq \mathcal{V}(p; \Lambda)/(\Lambda^2 + |m|^2)^2$  from (A.7). Since  $p^2 \leq \Lambda^2$  in Eq. (A.9), that is,  $\Lambda^2 - p^2/4 \geq (3/4)\Lambda^2$ , and  $\arcsin(|p|/(2\Lambda)) \geq |p|/(2\Lambda)$ , we have

$$\int \frac{d^4p}{(2\pi)^4} \frac{\Theta(\Lambda^2 - p^2)}{(p^2 + |m|^2)^2} \frac{(p+q)^2 \Theta(\Lambda^2 - (p+q)^2)}{(p+q)^2 + |m|^2} |\text{Eq. (A.5)}| \quad (\text{A.10})$$

$$\geq \frac{4 + 3\sqrt{3}}{128\pi^3} \frac{\Lambda^3}{(\Lambda^2 + |m|^2)^2} \int \frac{d^4p}{(2\pi)^4} \frac{\Theta(\Lambda^2 - p^2)}{(p^2 + |m|^2)^2} \frac{(p+q)^2 \Theta(\Lambda^2 - (p+q)^2)}{(p+q)^2 + |m|^2} |p| \\ \xrightarrow{\Lambda \rightarrow \infty} \frac{4 + 3\sqrt{3}}{1024\pi^5}. \quad (\text{A.11})$$

The last  $\Lambda \rightarrow \infty$  limit was found by setting  $m = 0$  and  $q = 0$ ; this is possible because this does not lead to the infrared divergence. The above relation shows that the contribution of Eq. (A.5) to Eq. (A.9) *does* survive even in the limit  $\Lambda \rightarrow \infty$ ; the contribution of the exotic term (A.5) in fact does not vanish in the two-loop level.

Nevertheless, it is still not clear whether this leads to a breakdown of locality. If the  $\Lambda \rightarrow \infty$  limit of Eq. (A.10) is *constant* in  $q$ , then the contribution could

be removed by a local counter term—a finite wave-function renormalization. On the other hand, if the limit of Eq. (A.10) is a nontrivial function such as  $\sim \ln(|q|/\Lambda)$ , the contribution cannot be removed by local counterterms and the expected locality is broken. Unfortunately, we have not been able to find which is really the case.

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